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Critical exponents from five-loop strong-coupling ϕ^4-theory in $4 - \varepsilon$ dimensions

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Abstract

With the help of strong-coupling theory, we calculate the critical exponents of O(N)-symmetric ϕ^4-theories in $4 - \varepsilon$ dimensions up to five loops with an accuracy comparable to that achieved by Borel-type resummation methods.

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1. Introduction

Recently, one of us [1, 2] has developed a new approach to critical exponents of field theories based on the strong-coupling limit of variational perturbation expansions [3] and [4][3]. This limit is relevant for critical phenomena if the renormalization constants are expressed in terms of the unrenormalized coupling constant. The theory was first applied successfully to O(N)-symmetric ϕ^4-theories in three dimensions yielding the three fundamental critical exponents ν, η, ω with high accuracy.

The method has also been shown to work for perturbation expansions of these theories in $4 - \varepsilon$ dimensions [5], but here only two-loop [5, 6] and three-loop [7] expansions were treated, where all results can be written down explicitly. In this paper we want to extend these calculations to the five-loop levels using the expansions given in [8, 9].

2. Resume of strong-coupling theory

From model studies of perturbation expansions of the anharmonic oscillator we have learned that variational perturbation expansions possess good strong-coupling limits [10] and [11][4], with a speed of convergence governed by the convergence radius of the strong-coupling expansion [12]. This has enabled us to set up a simple algorithm [4] for deriving uniformly convergent approximations to functions of which one knows a few initial Taylor coefficients

[3] Details of strong-coupling theory are found in chapter 5 of the textbook.

[4] This paper contains references to earlier, less powerful calculations of strong-coupling expansion coefficients from weak-coupling perturbation theory.

and an important scaling property: the functions approach a constant value with a given inverse power of the variable. The renormalized coupling constant g and the critical exponents of a ϕ^4-theory have precisely this property as a function of the bare coupling constant g_B. In $D = 4 - \varepsilon$ dimensions the approach is parametrized as follows [1]:

$$g(g_B) = g^* - \frac{\text{const}}{g_B^{\omega/\varepsilon}} + \cdots \tag{2.1}$$

where g^* is the infrared-stable fixed point, and ω is called the critical exponent of the approach to scaling. This exponent is universal, governing the approach to scaling of every function of $F(g)$,

$$f(g_B) = F(g(g_B)) = F(g^*) + F'(g^*) \times \frac{\text{const}}{g_B} \equiv f^* + \frac{\text{const}'}{g_B^{\omega/\varepsilon}}. \tag{2.2}$$

This type of scaling behaviour is observed experimentally in systems described by ϕ^4-theories, and strong-coupling theory is designed to calculate $f(g^*)$ and ω.

The difference between the expansions of field theory and of the harmonic operator lies mainly in the power sequence of the nonleading correction terms to (2.1). Whereas the harmonic oscillator has only corrections of the form $1/g_B^{n\omega}, n = 2, 3, 4, \ldots$, the field theory has also daughter corrections $1/g_B^{n\omega'}$ with $\omega' \neq \omega$. These will be neglected, this being equivalent to neglecting of confluent singularities at the infrared-stable fixed point in the renormalization group approach discussed by Nickel [13] and in [14].

Let $f(g_B)$ be a function with these properties, and suppose that we know its first $L + 1$ expansion terms,

$$f_L(g_B) = \sum_{l=0}^{L} a_l g_B^l. \tag{2.3}$$

More specifically than in equation (2.1), we assume that $f(g_B)$ approaches its constant strong-coupling limit f^* in the form of an inverse power series

$$f_M(g_B) = \sum_{m=0}^{M} b_m (g_B^{-2/q})^m \tag{2.4}$$

with a finite convergence radius [15]. Then the Lth approximation to the value f^* is obtained from the strong-coupling formula [1, 2, 5]

$$f_L^* = \operatorname*{opt}_{\hat{g}_B}\left[\sum_{l=0}^{L} a_l v_l \hat{g}_B^l\right] \qquad v_l \equiv \sum_{k=0}^{L-l} \binom{-ql/2}{k} (-1)^k. \tag{2.5}$$

The quantities v_l are simply binomial expansions of $(1-1)^{-ql/2}$ up to the order $L - l$. The expression in brackets has to be optimized in the variational parameter $\hat{g}_B$. The optimum is the smoothest among all real extrema. If there are no real extrema, the turning points serve the same purpose.

The derivation of this rule is simple: we replace g_B in (2.3) trivially by $\bar{g}_B \equiv g_B/\kappa^q$ with $\kappa = 1$. Then we rewrite, again trivially, κ^{-q} as $(K^2+\kappa^2-K^2)^{-q/2}$ with an arbitrary parameter K. Each term is now expanded in powers of $r = (\kappa^2 - K^2)/K^2$ assuming r to be of the order g_B. Then we take the limit $g_B \to \infty$ at a fixed ratio $\hat{g}_B \equiv g_B/K^q$, so that $K \to \infty$ like $g_B^{1/q}$ and $r \to -1$, yielding (2.5). Since the final result to all orders cannot depend on the arbitrary parameter K, we expect the best result to any finite order to be optimal at an extremal value of K, i.e. of $\hat{g}_B$.

The approach to the strong-coupling limit of r is $r = -1 + \kappa^2/K^2 = -1 + \mathrm{O}(g_B^{-2/q})$. This implies the leading correction to f_L^* to be of the order of $g_B^{-2/q}$. Application of the theory to a function with the strong-coupling behaviour (2.1) requires therefore a parameter $q = 2\varepsilon/\omega$ in formula (2.5). A systematic expansion in powers of K^2 leads to the strong-coupling expansion (2.4).

3. Renormalization constants and critical exponents

Let us briefly recall the definitions of the ϕ^4-theory in $D = 4-\varepsilon$ dimensions whose five-loop expansions we want to evaluate. The bare Euclidean action is

$$\mathcal{A} = \int \mathrm{d}^D x \left\{ \frac{1}{2}[\partial\phi_{\mathrm{B}}(x)]^2 + \frac{1}{2}m_{\mathrm{B}}^2\phi_{\mathrm{B}}^2(x) + (4\pi)^2\frac{g_{\mathrm{B}}}{4!}[\phi_{\mathrm{B}}^2(x)]^2 \right\} \tag{3.1}$$

where the field $\phi_{\mathrm{B}}(x)$ is an N-dimensional vector, the action being O(N)-symmetric. The Ising model corresponds to $N = 1$, the superfluid phase transition to $N = 2$, the classical Heisenberg magnet to $N = 3$. The critical behaviour of dilute polymer solutions is described by $N = 0$.

By calculating the Feynman integrals, regularized via an expansion in $\varepsilon = 4 - D$ and arbitrary mass scale μ, one obtains renormalized values of mass, coupling constant and field related to the bare quantities by renormalization constants Z_ϕ, Z_m, Z_g:

$$m_{\mathrm{B}}^2 = m^2\, Z_m Z_\phi^{-1} \qquad g_{\mathrm{B}} = g\, Z_g Z_\phi^{-2} \qquad \phi_{\mathrm{B}} = \phi\, Z_\phi^{1/2}. \tag{3.2}$$

Up to two loops, perturbation theory yields the following expansions in powers of the dimensionless reduced coupling constant $g_{\mathrm{B}} \equiv \lambda_{\mathrm{B}}/\mu^{\varepsilon}$:

$$g = g_{\mathrm{B}} - \frac{N+8}{3\varepsilon}g_{\mathrm{B}}^2 + \left[\frac{(N+8)^2}{9\varepsilon^2} + \frac{3N+14}{6\varepsilon}\right]g_{\mathrm{B}}^3 + \cdots \tag{3.3}$$

$$\frac{m^2}{m_{\mathrm{B}}^2} = 1 - \frac{N+2}{3}\frac{g_{\mathrm{B}}}{\varepsilon} + \frac{N+2}{9}\left[\frac{N+5}{\varepsilon^2} + \frac{5}{4\varepsilon}\right]g_{\mathrm{B}}^2 + \cdots \tag{3.4}$$

$$\frac{\phi^2}{\phi_{\mathrm{B}}^2} = 1 + \frac{N+2}{36}\frac{g_{\mathrm{B}}^2}{\varepsilon} + \cdots. \tag{3.5}$$

We refrain from writing down the lengthy five-loop expressions calculated in [8], since they can be downloaded from the internet [9]. We now set the scale parameter μ equal to the physical mass m and consider all quantities as functions of $g_{\mathrm{B}} = \lambda_{\mathrm{B}}/m^{\varepsilon}$. In order to describe second-order phase transitions, we let m_{B}^2 go to zero like $\tau = \text{const} \times (T - T_{\mathrm{c}})$ as the temperature T approaches the critical temperature T_{c}, and assume that also m^2 goes to zero, and thus g_{B} to infinity. The latter assumption will be seen to be self-consistent at the end. Assuming the theory to scale as suggested by experiments, we now determine the value of the renormalized coupling constant g and of the exponent ω of approach in the strong-coupling limit $g_{\mathrm{B}} \to \infty$, assuming the behaviour (2.1). First we apply formula (2.5) to the logarithmic derivative (3.6) of the function $g(g_{\mathrm{B}})$:

$$s(g_{\mathrm{B}}) \equiv g_{\mathrm{B}}g'(g_{\mathrm{B}})/g(g_{\mathrm{B}}). \tag{3.6}$$

Setting $s_L^* = 0$ determines the approximation ω_L to ω.

The other critical exponents are found as follows. From the experimental behaviour of systems described by ϕ^4-theories, we know that the ratios m^2/m_{B}^2 and $\phi^2/\phi_{\mathrm{B}}^2$ have a limiting power behaviour for small m:

$$\frac{m^2}{m_{\mathrm{B}}^2} \propto g_{\mathrm{B}}^{-\eta_m/\varepsilon} \propto m^{\eta_m} \qquad \frac{\phi^2}{\phi_{\mathrm{B}}^2} \propto g_{\mathrm{B}}^{\eta/\varepsilon} \propto m^{-\eta}. \tag{3.7}$$

The powers η_m and η can then be calculated from the strong-coupling limits of the logarithmic derivatives

$$\eta_m(g_B) = -\varepsilon \frac{d}{d \log g_B} \log \frac{m^2}{m_B^2} \qquad \eta(g_B) = \varepsilon \frac{d}{d \log g_B} \log \frac{\phi^2}{\phi_B^2}. \tag{3.8}$$

Inserting (3.4) and (3.5) on the right-hand sides yields the expansions

$$\eta_m(g_B) = \frac{N+2}{3} g_B - \frac{N+2}{18}\left(5 + 2\frac{N+8}{\varepsilon}\right) g_B^2 + \cdots \tag{3.9}$$

$$\eta(g_B) = \frac{N+2}{18} g_B^2 + \cdots. \tag{3.10}$$

When approaching the second-order phase transitions where the bare mass m_B^2 vanishes like τ, the physical mass m^2 vanishes with a different power of τ. This power is obtained from the first equation in (3.7), which shows that $m \propto \tau^{1/(2-\eta_m)}$. In experiments one observes that the coherence length of fluctuations $\xi = 1/m$ increases near T_c like $\tau^{-\nu}$. Comparison with the previous equation shows that the critical exponent ν is equal to $1/(2 - \eta_m)$. Similarly we see from the second equation in (3.7) that the scaling dimension $D/2 - 1$ of the free field ϕ_B for $T \to T_c$ is changed in the strong-coupling limit to $D/2 - 1 + \eta/2$, the number η being the so-called anomalous dimension of the field. This implies a change in the large-distance behaviour of the correlation functions $\langle \phi(x)\phi(0)\rangle$ at T_c from the free-field behaviour r^{-D+2} to $r^{-D+2-\eta}$.

The magnetic susceptibility is determined by the integrated correlation function $\langle \phi_B(x)\phi_B(0)\rangle$. At zero coupling constant g_B, this is proportional to $1/m_B^2 \propto \tau^{-1}$. The interaction changes this to $m^{-2}\phi_B^2/\phi^2$. This quantity has a temperature behaviour $m^{-(2-\eta)} \propto \tau^{-\nu(2-\eta)} \equiv \tau^{-\gamma}$, which defines the critical exponent $\gamma = \nu(2 - \eta)$ governing the divergence of the susceptibility line. Using $\nu = 1/(2 - \eta_m)$ and the expansions (3.9), (3.10), we obtain for $\gamma(g_B)$ the perturbation expansion up to second order in g_B:

$$\gamma(g_B) = 1 + \frac{N+2}{6} g_B + \frac{N+2}{36}\left(N - 4 - 2\frac{N+8}{\varepsilon}\right) g_B^2 + \cdots. \tag{3.11}$$

Explicit two-loop results were given in [5] and [6] and will not be repeated here. We only mention that they lead to resummed expressions for the same ε-expansions as found by renormalization group techniques.

4. Five-loop results

We now extend the two-loop results of [5,6] to five loops using the power series for the critical exponents of [8, 9]. In a first step, we determine the parameter ω for which the logarithmic derivative of $g(g_B)$ approaches zero for $g_B \to \infty$. We therefore insert the coefficients of the power series of the logarithmic derivative of $s(g_B)$ from equation (3.6) into (2.5) and determine $q = 2/\omega$ for $L = 2, 3, 4, 5$, to make $s_L^* = 0$. The resulting ε-expansions for the approach-to-scaling parameter ω reproduce the well known ε-expansions in [8] up to the corresponding order. In figure 1, the approximations ω_L are plotted against the number of loops L for $\varepsilon = 1$.

Apparently, the five-loop results are still some distance away from a constant ($L \to \infty$)-limit. The slow approach to the limit calls for a suitable extrapolation method. The general convergence behaviour in the limit $L \to \infty$ was determined in [1] to be of the general form

$$f^*(L) \approx f^* + \text{const} \times e^{-c L^{1-\omega}}. \tag{4.1}$$

We therefore plot the approximations s_L for a given ω near the expected critical exponent against L. To exploit the knowledge of the behaviour (4.1) we fit the points by the theoretical

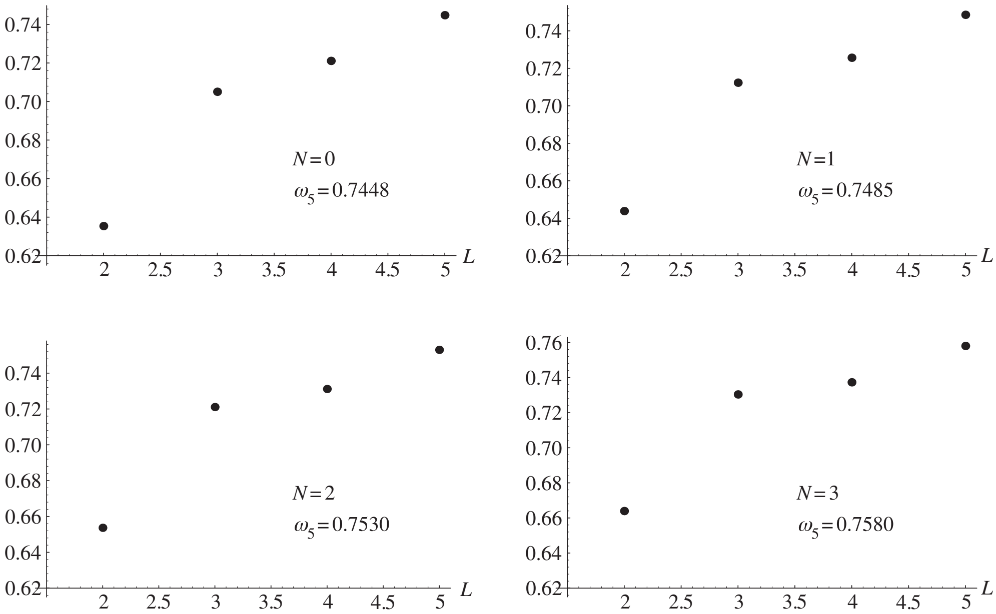

Figure 1. Critical exponent of approach to scaling ω calculated from $s_L^* = 0$, plotted against the order of approximation L.

curve (4.1) to determine the limit s^*. Then ω is varied, and the plots are repeated until s^* is zero. The resulting value for ω gives the desired critical exponent, and the associated plots are shown in figure 2. Since the optimal variational parameter $\hat{g}_\mathrm{B}$ is determined from minima and turning points for even and odd approximants in alternate order, the points are best fitted by two different curves.

In order to determine the common constant c one plots even and odd approximations s_L directly against the variable $x_L = \mathrm{e}^{-cL^{1-\omega}}$. The constant c is then used to fit straight lines through even and odd approximations which cross at zero x_L. This procedure is shown in figure 2, and yields the curve shown in figure 3. The resulting ω-values are listed in table 1. They will now be used to derive the strong-coupling limits for the exponents ν, γ and η.

4.1. Exponents ν

For the calculation of the critical exponent ν, we proceed in two different ways. This will give us an idea of the systematic error of the method. First we find the five-loop expansions for $\nu(g_\mathrm{B})$ using the relation $\nu(g_\mathrm{B}) = 1/[2 - \eta_m(g_\mathrm{B})]$. From this we calculate their strong-coupling approximation ν_L for $L = 2, 3, 4, 5$. After extrapolating these to infinite L, we obtain the numbers listed for different universality classes O(N) in table 1 under the heading (I). The corresponding extrapolation fits are plotted in figures 4 and 5. The resulting values for the critical exponent $\nu(\infty)$ are indicated by horizontal lines in figure 5.

The second method proceeds by calculating the strong-coupling values of $\eta_m(g_\mathrm{B})$ for $L = 2, 3, 4, 5$. After extrapolating these to infinite L, the critical exponent ν is found from $\nu = 1/(2 - \eta_m^*)$. The results are listed in table 1 under the heading (II). The table shows in parentheses the $L = 5$-approximation for each quantity, from which we see how far away the extrapolated result is from the highest approximation.

By repeating all calculations for a slightly different ω-value, we deduce the dependence

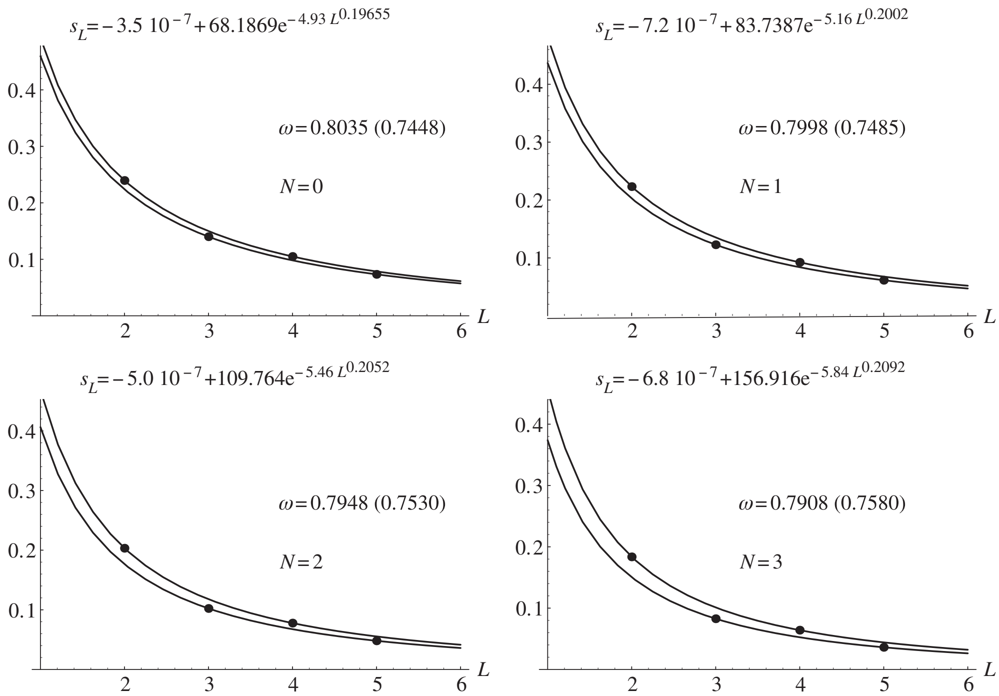

Figure 2. Extrapolation of the solutions of the equation $s_L^* = 0$ to $L \to \infty$ with the help of the theoretically expected large-L behaviour (4.1). The value of ω where s_L^* goes to zero for $L \to \infty$ determines the critical exponent $\omega = 2/q$. The best extrapolating function is written at the top of the figure.

of our results on the critical exponent ω used in the resummation process:

$$\Delta\nu = \left\{ \begin{array}{c} -0.0900 \times (\omega - 0.8035) \\ -0.1375 \times (\omega - 0.7998) \\ -0.1853 \times (\omega - 0.7948) \\ -0.2271 \times (\omega - 0.7908) \end{array} \right\} \qquad \text{for} \qquad \left\{ \begin{array}{c} N = 0 \\ N = 1 \\ N = 2 \\ N = 3 \end{array} \right\}. \tag{4.2}$$

4.2. Exponents η and γ

The calculation of the critical exponent η is difficult in all resummation schemes since the power series of $\eta(g_B)$ starts out with g_B^2, so there is one approximation less than for ν. The three approximations η_3, η_4, η_5 obtained from the five-loop expansions are not sufficient to carry out the above extrapolation procedure. The exponent is therefore calculated from the strong-coupling limit of the power series for $\bar{\eta}(g_B) \equiv \eta_m(g_B) + \eta(g_B)$, which supplies us with the combination of critical exponents $2-1/\nu+\eta$. After finding $\bar{\eta}^*$ we subtract from this $2-1/\nu$ and obtain the desired η. If we use ν (I) of table 1 in this subtraction, we obtain η-values listed as η (I) in table 1. From ν (II) we obtain η (II). The fits leading to the strong-coupling limits of $\bar{\eta}(g_B)$ are shown in figures 6 and 7. As before, the limiting values for $L \to \infty$ are indicated by horizontal lines. The fitted extrapolation function is displayed at the top of each figure.

An independent strong-coupling calculation for the critical exponent η may be obtained by resumming the series expansion for the critical exponent of the susceptibility $\gamma = \nu(2-\eta)$. The extrapolation plots for this exponent are shown in figures 8 and 9. The resulting value for

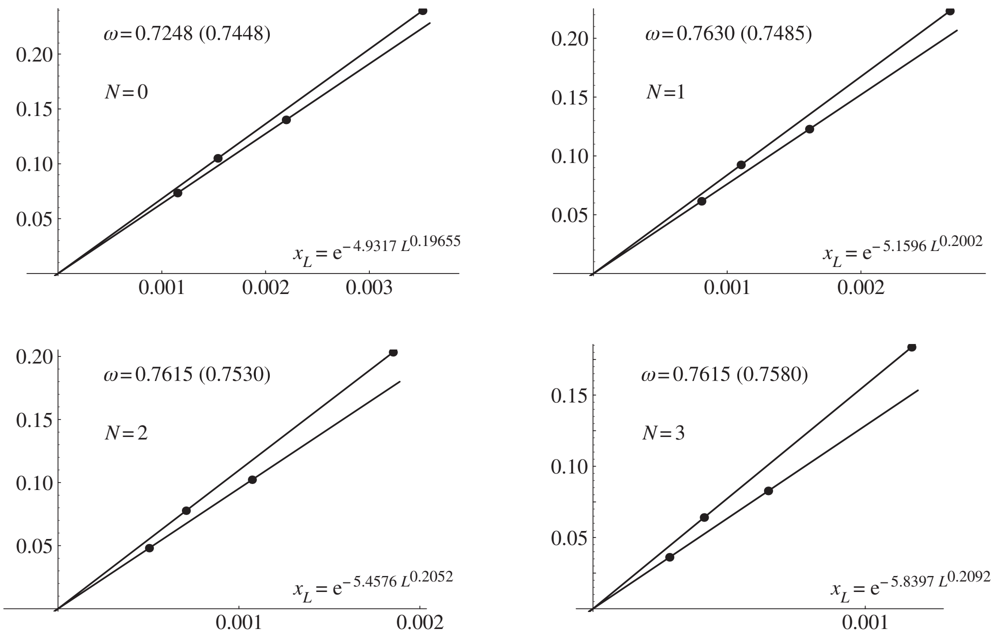

Figure 3. The same plot as in figure 2, but against the variable $x_L = e^{-cL^{1-\omega}}$. The parameter c is fixed by requiring the straight lines to cross on the vertical axis. When this intercept lies at the origin, we have found the critical exponent ω, written at the top of each plot. For comparison, we also show a direct plot against L in figure 2.

γ is also contained in table 1. As in all entries, we have listed the fifth-order approximations in parentheses to illustrate the extrapolation distance to infinite order L.

The dependence on the value of ω is of the same order of magnitude as for ν:

$$\Delta\gamma = \begin{Bmatrix} -0.1500 \times (\omega - 0.8035) \\ -0.2237 \times (\omega - 0.7998) \\ -0.3147 \times (\omega - 0.7948) \\ -0.4014 \times (\omega - 0.7908) \end{Bmatrix} \qquad \text{for} \quad \begin{Bmatrix} N=0 \\ N=1 \\ N=2 \\ N=3 \end{Bmatrix}. \tag{4.3}$$

4.3. Comparison with previous results and experiments

In table 1 we have added to our results also those obtained by other methods. Since an extensive table has been published before (table 4 in [2]), we confine ourselves here to results of the resummation of the ε-expansion by Guida and Zinn-Justin in [18], and those from three-dimensional variational perturbation theory to sixth order for ω and to order seven for ν and η in [2]. The difference between ν (I) and ν (II), and η (I) and η (II) is considerably smaller than the typical errors in the other references.

The results of our strong-coupling theory agree very well with those obtained from Borel-type resummation although we do not make use of the known large-order behaviour. For a good test of the reliability of our results we compare our results with experiments. The most precise experimental values are available from specific heat measurements performed on superfluid helium near the λ-point at zero gravity in the space shuttle in 1992, which are reported in [16][5].

[5] The initially published fit to the data in the first paper of [16] was erroneous and corrected in the second paper of [16].

Table 1. Extrapolated exponents of five-loop strong-coupling theory and comparison with the results from Borel-type resummation of [18] (GZ) and [22] (MN), and from variational perturbation theory in $D = 3$ dimensions (VPT, $D = 3$) of [2]. The parentheses behind each number show the five-loop approximation to see the extrapolation distance. The two values for ν come one from a resummation of the series for ν itself (I) and the other from the series for ν^{-1} (II). The two values for η come from subtracting for one the value ν (I) and for the other the value ν (II).

	VPT, $D = 4 - \varepsilon$		Borel res. (GZ)	VPT, $D = 3$	MN, $D = 3$
	$\omega(\omega_5)$				
$N = 0$	0.803 5(0.7448)		0.828 ± 0.023	0.810	
$N = 1$	0.799 8(0.7485)		0.814 ± 0.018	0.805	
$N = 2$	0.794 8(0.7530)		0.802 ± 0.018	0.800	
$N = 3$	0.790 8(0.7580)		0.794 ± 0.018	0.797	
	$\nu(\nu_5)$ (I)	$\nu(\nu_5)$ (II)			
$N = 0$	0.5874(0.5809)	0.5878(0.5832)	0.5875 ± 0.0018	0.588 3	0.5872 ± 0.0004
$N = 1$	0.6292(0.6171)	0.6294(0.6222)	0.6293 ± 0.0026	0.630 5	0.6301 ± 0.0005
$N = 2$	0.6697(0.6509)	0.6692(0.6597)	0.6685 ± 0.0040	0.671 0	0.6715 ± 0.0007
$N = 3$	0.7081(0.6821)	0.7063(0.6951)	0.7050 ± 0.0055	0.707 5	0.7096 ± 0.0008
	$\eta(\eta_5)$ (I)	$\eta(\eta_5)$ (II)			
$N = 0$	0.0316(0.0234)	0.0305(0.0234)	0.0300 ± 0.0060	0.032 15	0.0297 ± 0.0009
$N = 1$	0.0373(0.0308)	0.0367(0.0308)	0.0360 ± 0.0060	0.033 70	0.0355 ± 0.0009
$N = 2$	0.0396(0.0365)	0.0396(0.0365)	0.0385 ± 0.0065	0.034 80	0.0377 ± 0.0006
$N = 3$	0.0367(0.0409)	0.0402(0.0409)	0.0380 ± 0.0060	0.034 47	0.0374 ± 0.0004
	$\gamma(\gamma_5)$				
$N = 0$	1.157 6(1.1503)		1.1575 ± 0.0050	1.616	1.1569 ± 0.0004
$N = 1$	1.234 9(1.2194)		1.2360 ± 0.0040	1.241	1.2378 ± 0.0006
$N = 2$	1.310 45(1.2846)		1.3120 ± 0.0085	1.318	1.3178 ± 0.0010
$N = 3$	1.383 0(1.3452)		1.3830 ± 0.0135	1.390	1.3926 ± 0.0010

A best fit through the data points yields for the essential exponent $\alpha = 2 - 3\nu$ the value

$$\alpha = -0.010\,56 \pm 0.000\,38 \tag{4.4}$$

corresponding to

$$\nu = 0.670\,95 \pm 0.000\,13. \tag{4.5}$$

Our resummation results in table 1 imply a value

$$\nu_{\text{ours}} = 0.6697 \pm 0.0013 \tag{4.6}$$

corresponding to

$$\alpha_{\text{ours}} = -0.0091 \pm 0.0039. \tag{4.7}$$

This agrees satisfactorily with the experimental result. In figure 10 we have compared our result with other experiments and various theoretical determinations.

Some remarks are necessary on the error estimates of our result. They are purely based on an inspection of the way the extrapolation curves to infinite order in figures 1, 2 converge against their limits. No systematic errors have been taken into account. These arise principally from the fact that if one works with different functions of the critical exponents rather than the exponents themselves, the associated series could give quite different results. In fact, it is possible to set up some peculiar function of any of the exponents, whose power series expansion has coefficients which do not render any strong-coupling limit. This freedom is ignored, and it is so in all previous resummation procedures for critical exponents.

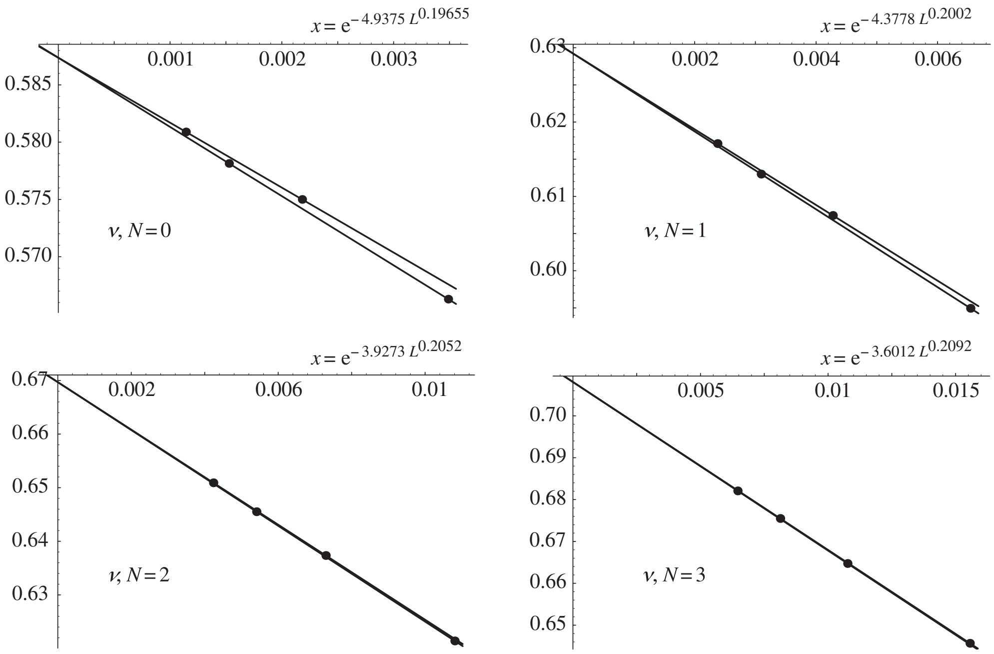

Figure 4. Critical exponent ν_L (I) obtained from variational perturbation theory plotted as a function of x_L. Requiring the lines to cross at $x_L = 0$ determines the parameter c in x_L. See the text.

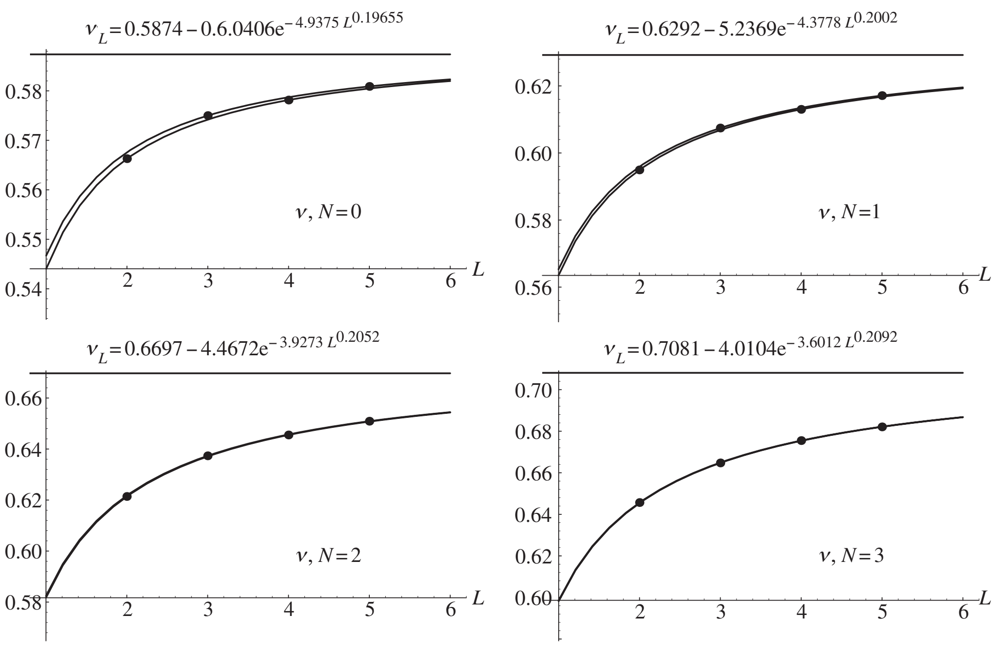

Figure 5. The same plot as in figure 4, but against L. The fit function is written at the top of the figure.

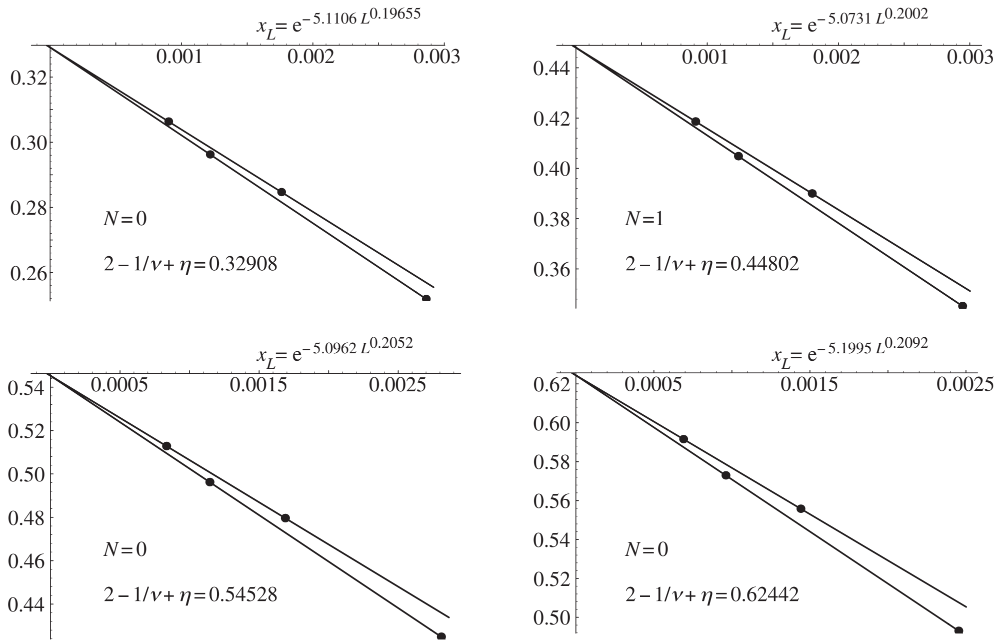

Figure 6. Determination of the critical exponent η from the strong-coupling limit of $\eta_m + \eta$ plotted as a function of x_L. Requiring the lines to cross at $x_L = 0$ determines the parameter c in x_L. See the text.

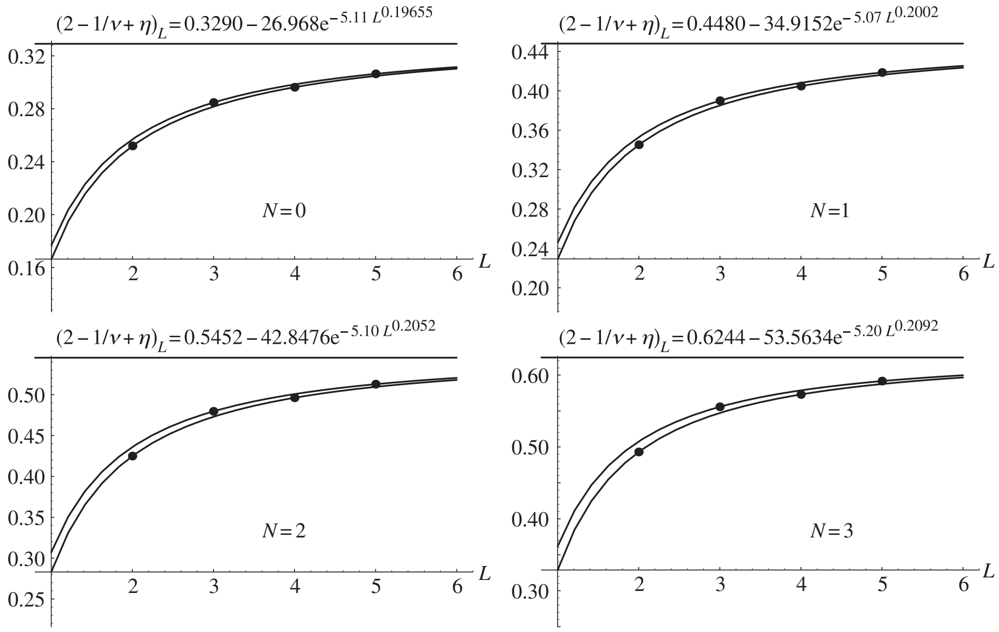

Figure 7. The same as above, plotted against the order of approximation L and the fit function written at the top of the figure.

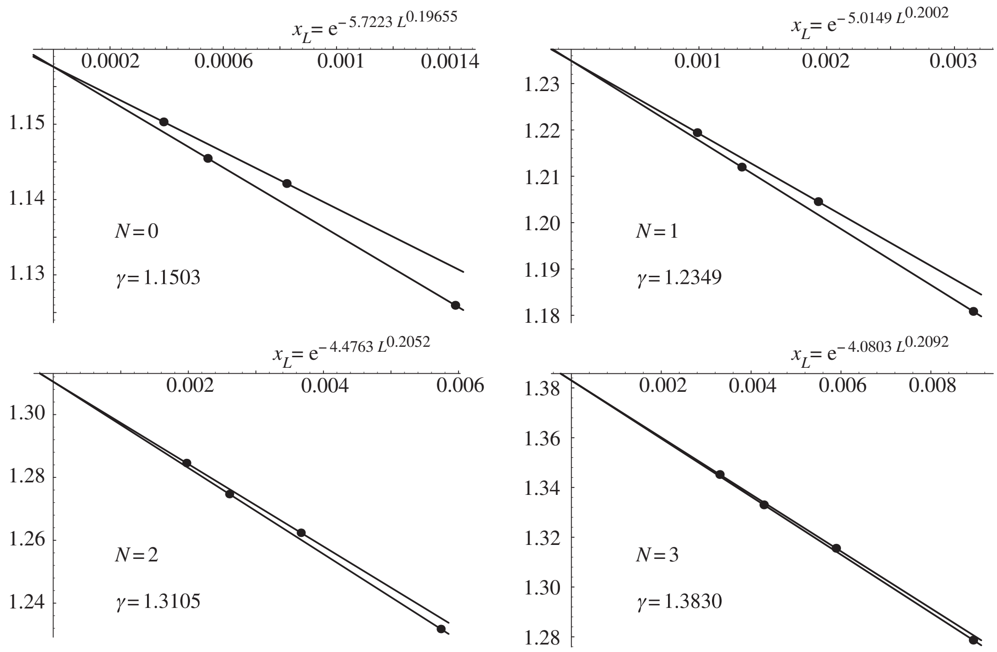

Figure 8. Critical exponent γ obtained from variational perturbation theory plotted as a function of x_L. Requiring the lines to cross at $x_L = 0$ determines the parameter c in x_L. See the text.

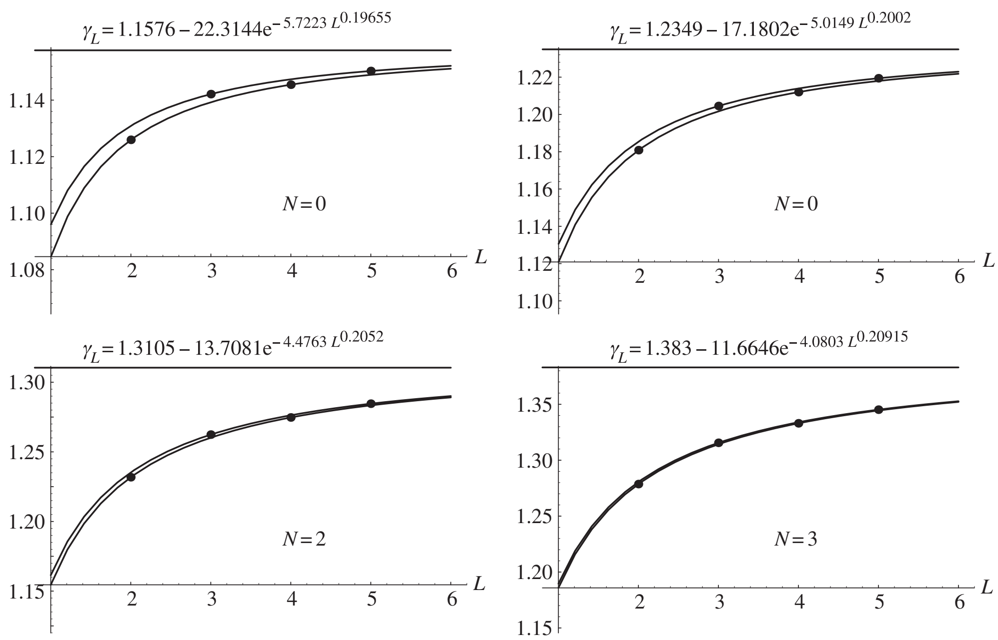

Figure 9. The same plot as in figure 8, but against L. The fit function is written at the top of the figure.

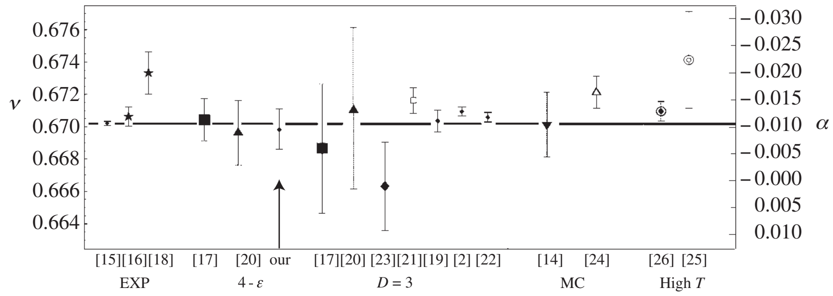

Figure 10. Critical exponent ν in comparison with experimental data and results from other resummation schemes.

4.4. Conclusion

Application of strong-coupling theory to five-loop perturbation expansions of $O(N)$-symmetric ϕ^4-theories in $4-\varepsilon$ dimensions yields satisfactory values for all critical exponents, and a very good agreement of the exponent α with the experimental space shuttle data as a test of the reliability of our calculation method. More details can be found in a forthcoming book on this subject [28].

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